Theory of Computation
Problem Set 4
Universidad Politecnica de San Luis Potosi

Please start solving these problems immediately, don't procrastinate, and work in study groups.

Please do not simply copy answers that you do not fully understand;

Advice: Please try to solve the easier problems first (where the meta-problem here is to figure out which are the easier ones ☺). Don't spend too long on any single problem without also attempting (in parallel) to solve other problems as well. This way, solutions to the easier problems (at least easier for you) will reveal themselves much sooner (think about this as a "hedging strategy" or "dovetailing strategy").

1. Solve the following problems from the [Sipser, Second Edition] textbook: USE JFLAP for design

   Pages 84 -8 5: 1.5, 1.6, 1.7 and 1.8

2. Define a new operation ♣ on languages as follows: ♣(L) = $\{w \mid \exists\, w\, \Sigma \in^{*}, ww^{R} \in L\}$, where $w^{R=}$ denotes the "reverse" of the string w. Does ♣ preserve regularity?

3. Are there two non-regular languages whose concatenation is regular? Are there a countably infinite number of such examples? Are there an uncountable number of such examples?

4. Define a new operation ♦ on languages as follows: ♦(L) = $\{w \mid \exists\, z \ni {}^{*}\Sigma \in |w|=|z| \wedge wz \in L\}$. Does the operation ♦ preserve regularity?

5. We define the SHUFFLE of two strings v,w $\Sigma \in^{*}$ as:

   SHUFFLE(v,w) = $\{v_1w_1v_2w_2...v_kw_k \mid v=v_1v_2...v_k, w=w_1w_2...w_k,$
   $\text{and for some } k \geq 1, v_i,w_i\ \Sigma \in^{*}, 1 \leq i \leq k\}$

   For example, 212**ab**1**baa**2**b**22∈ SHUFFLE(**abbaab**,2121222)

   Extend the definition of SHUFFLE to two languages $L_1,L_2 \subseteq \Sigma^{*}$ as follows:

   SHUFFLE(L_1,L_2) = $\{w \mid w_1 \in L_1, w_2 \in L_2, w \in$ SHUFFLE$(w_1,w_2)\}$

   a) Is the SHUFFLE of two finite languages necessarily finite?

   b) Is the SHUFFLE of two regular languages necessarily regular?

6. Define a DIVISION operator on languages as follows:

$$\frac{L_1}{L_2} = \{w \mid w\ \Sigma \in^{*} \text{ and } \exists\, v \in L_2 \ni\ wv \in L_1\}$$

   Does DIVISION preserve regularity? What if L_1 is regular and L_2 is arbitrary?

7. Which of the following modifications / restrictions to finite automata would change the class of languages accepted relative to "normal" (unmodified) finite automata? (Assume a fixed alphabet of say $\Sigma = \{a,b\}$)

a) The ability to move the read head backwards (as well as forwards) on the input.

b) The ability to write on (as well as read from) the input tape.

c) Both a) and b) simultaneously.

d) Having 2 read-heads moving (independently, left-to-right) over the input.

e) Having no more than one billion different states.

8. Determine whether each of the following languages is regular:

a) $\{a^n a^n a^n \mid n>0\}$

b) $\{www \mid w \in \{x,y,z\}^*, |w| < 10^{100}\}$

c) $\{vw \mid v,w \in \{a,b\}^*\}$

d) $\{ww \mid w \in \{a\}^*\}$

9. Are there two non-finitely-describable languages whose concatenation is regular? Are there a= countably infinite number of such examples? Are there an uncountable number of such= examples?

10. Let F denote some finite language, R denote some regular language, C denote some context-free language, and N denote some non-context-free languages. For each one of the following statements, prove whether it is always true, sometimes true, or never true:

a) RC is regular

b) N – R is regular

c) $N \cap F$ is not regular

d) N – F is regular

e) C^* is infinite

f) R is context-free

g) $R \cup C$ is finite

h) C is regular

i) N is infinite

11. Give algorithms (i.e., a well-defined, deterministic, always-terminating decision procedures, and state their time complexities) to determine whether for a given finite automaton M, L(M) is:

a) countable

b) empty

c) Σ^*

d) finite

e) infinite

f) co-finite (i.e., with a finite complement)

g) regular

h) context-free

i) also accepted by a smaller FA (i.e., with fewer states than M)

12. Give algorithms (and state their time complexities) to determine whether for a given pair of finite automata:

a) they both accept the same language

b) the intersection of their languages is empty

c) the intersection of their languages is finite

d) the union of their languages is finite

e) the intersection of their languages is infinite

f) the union of their languages is infinite

g) the intersection of their languages is Σ^*

h) the difference of their languages is finite

13. Let $L=\{0^n1^n \mid n\geq 0\}$. Is $\overline{L}$ (i.e. the complement of L) a regular language?

14. Let $L= \{0^i1^j \mid i \neq j\}$. Is L a regular language?